dash optimization

State-of-the-Optimization using Xpress-MP v2006

INFORMS Annual Meeting
Pittsburgh, USA
November 5–8, 2006

by Alkis Vazacopoulos

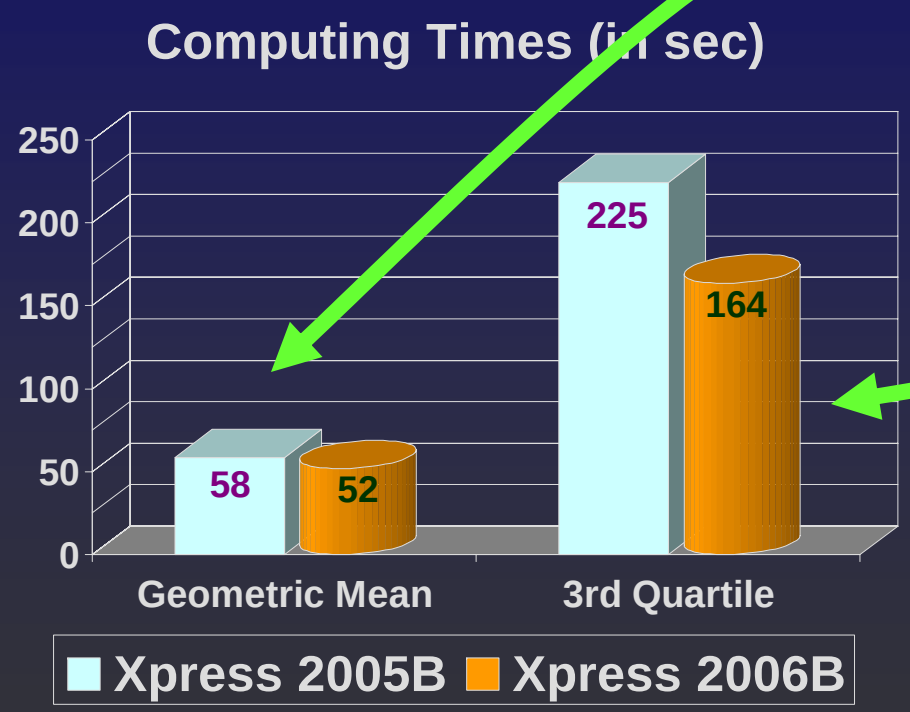
dash optimization

Outline

- LP benchmarks
- Xpress performance on MIPLIB 2003
- Conclusions

Barrier Algorithm

(Mittelmann Benchmark for LP)



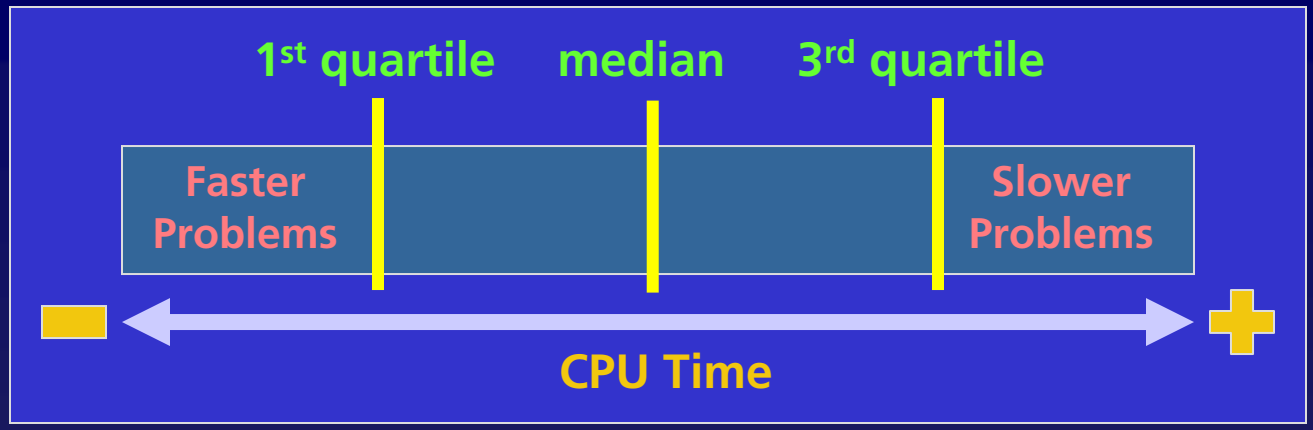
2006B is 12% faster than 2005B on an "average" case

2006B is 27% faster than 2005B on the "larger" problems

NOTE: The computing times include the (crossover) time needed to find a basic feasible solution. Considers 35 LPs.

dash optimization

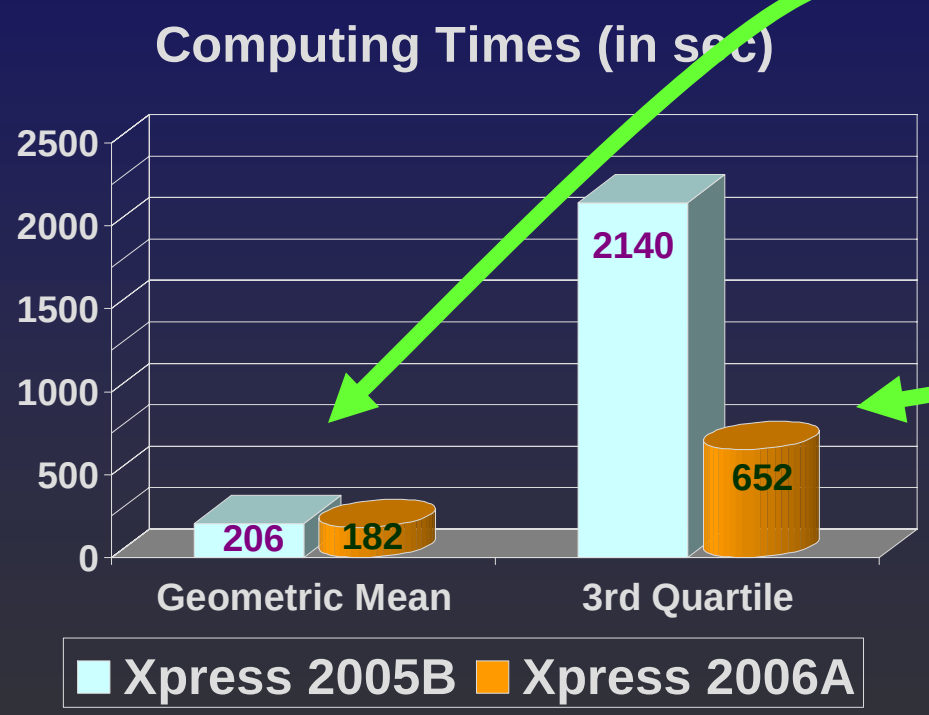
Quartile



- A **quartile** is any of the three values which divide the **sorted** data set into four equal parts, so that each part represents 1/4th of the sample
  - **1st quartile**
    - **lower quartile** = cuts off lowest 25% of data = *25th percentile*
  - **2nd quartile**
    - ***median*** = cuts data set in half = *50th percentile*
  - **3rd quartile**
    - **upper quartile** = cuts off highest 25% of data, or lowest 75% = *75th percentile*

Dual Algorithm

(Mittelmann Benchmark for LP)



2006B is 12% faster than 2005B on an "average" case

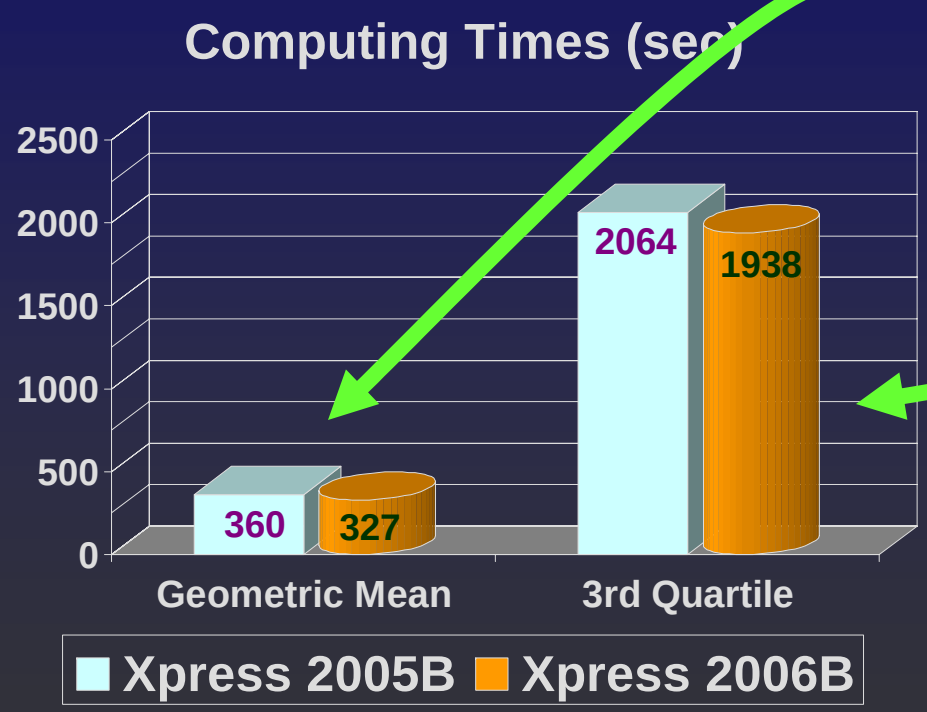
2006B is 70% faster than 2005B on the "larger" problems

NOTE: Considers 34 LPs.

dash optimization

Primal Algorithm

(Mittelmann Benchmark for LP)



2006B is 11% faster than 2005B on an "average" case

2006B is 7% faster than 2005B on the "larger" problems

NOTE: Considers 34 LPs.

dash optimization

Xpress LP Improvements

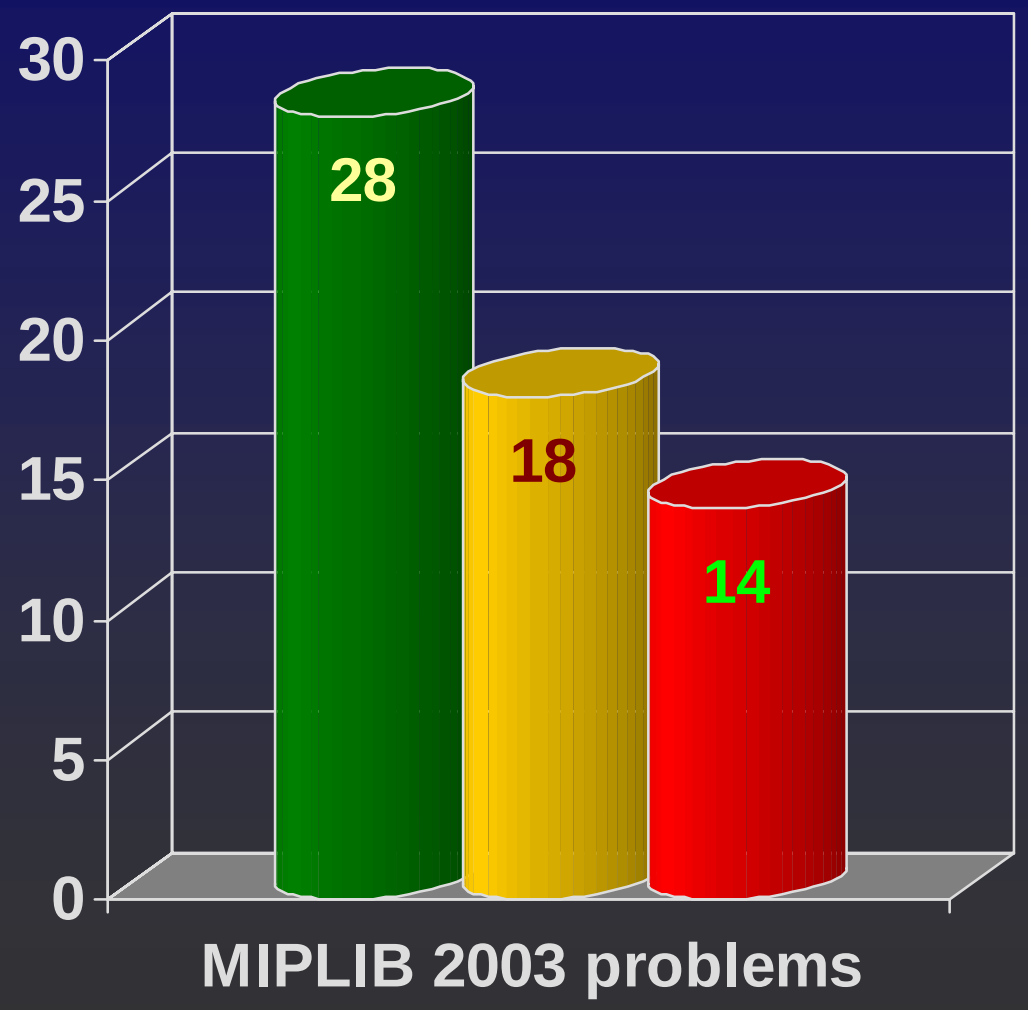
(Mittelmann Benchmark for LP)

LP Algorithms	2006B Improvements from 2005B	
	Geometric Mean	***3rd Quartile***
	Computing Times	
Barrier	12% faster	27% faster
Dual	12% faster	70% faster
Primal	11% faster	7% faster

dash optimization

MIPLIB 2003

MIPLIB 2003
consists of 60 MIPs.



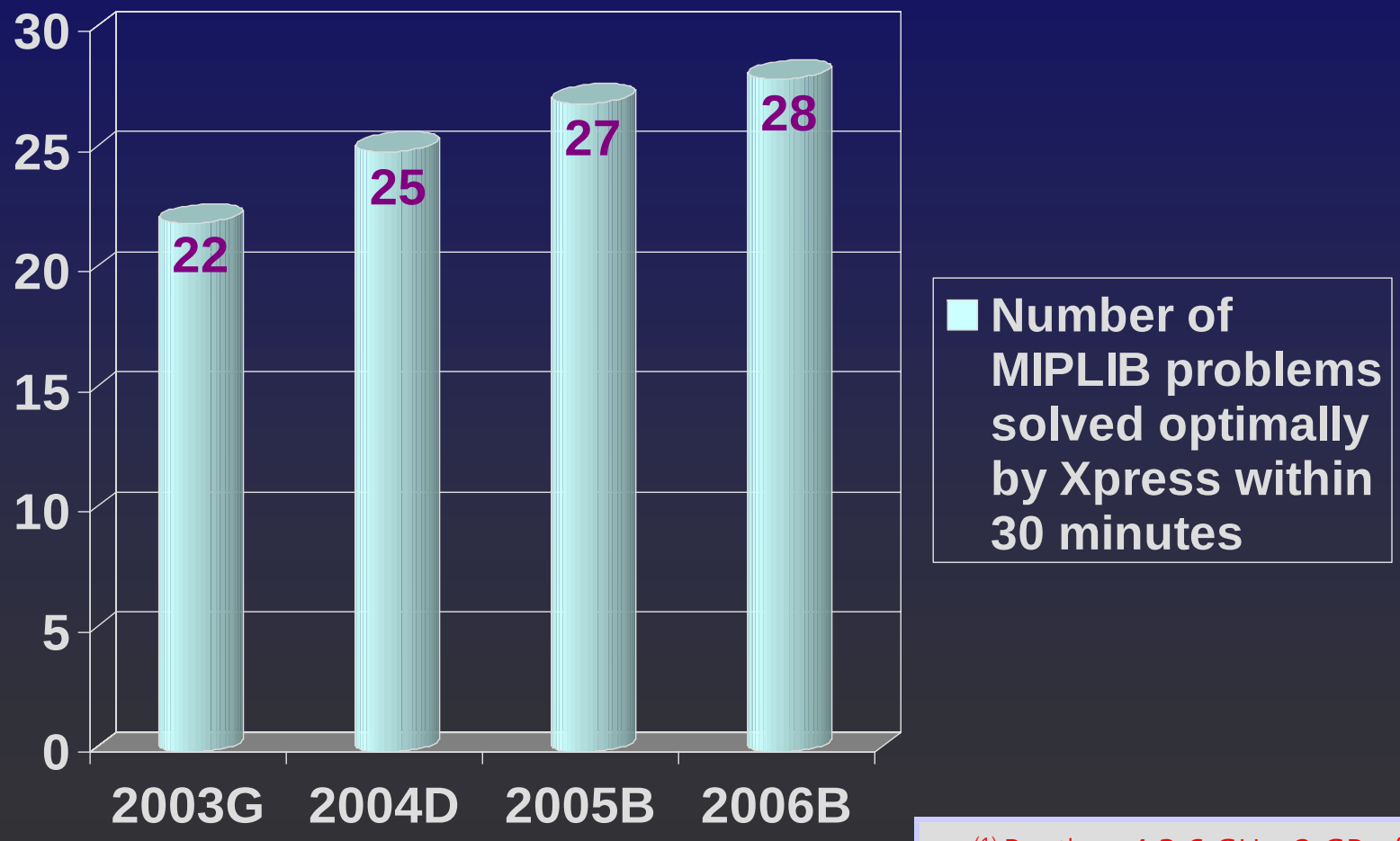
According to the MIPLIB website (Oct-2006):

- **Solved within 1 hour with a Commercial Solver**
- **Optimum is known, but not in the previous conditions**
- **Optimum is unknown**

Xpress Performance on MIPLIB 2003

- Xpress Improvements in the Group of "Simpler" Problems
- Xpress Improvements in the Group of "Challenging" Problems

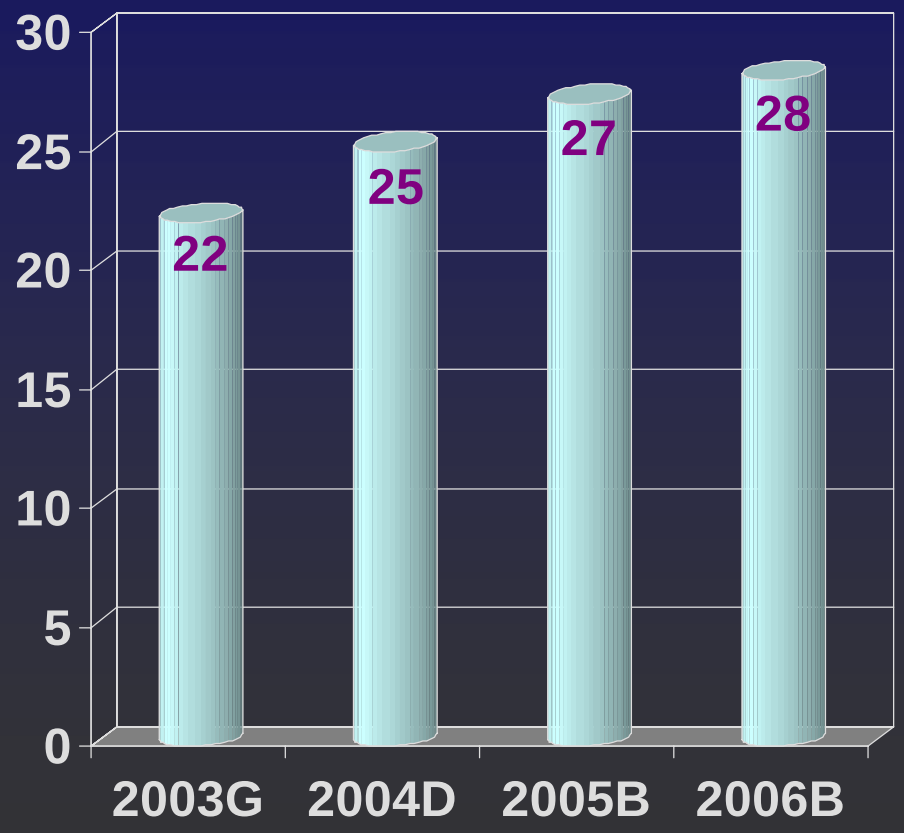
MIPLIB 2003: Optimum within 30' with Default Settings



(1) Pentium 4 3.6 GHz, 2 GB of RAM, Hyper-Threading, Windows XP

dash optimization

MIPLIB 2003: Optimum within 30' with Default Settings



In 30 min, **Xpress 2006B** solves **27 "simple" problems** (according to the MIPLIB website)

+

harp2,

which can be solved by Xpress 2006B in **1 minute**[1] with default settings

[1] Pentium 4 3.6 GHz, 2 GB of RAM, Hyper-Threading, Windows XP

dash optimization

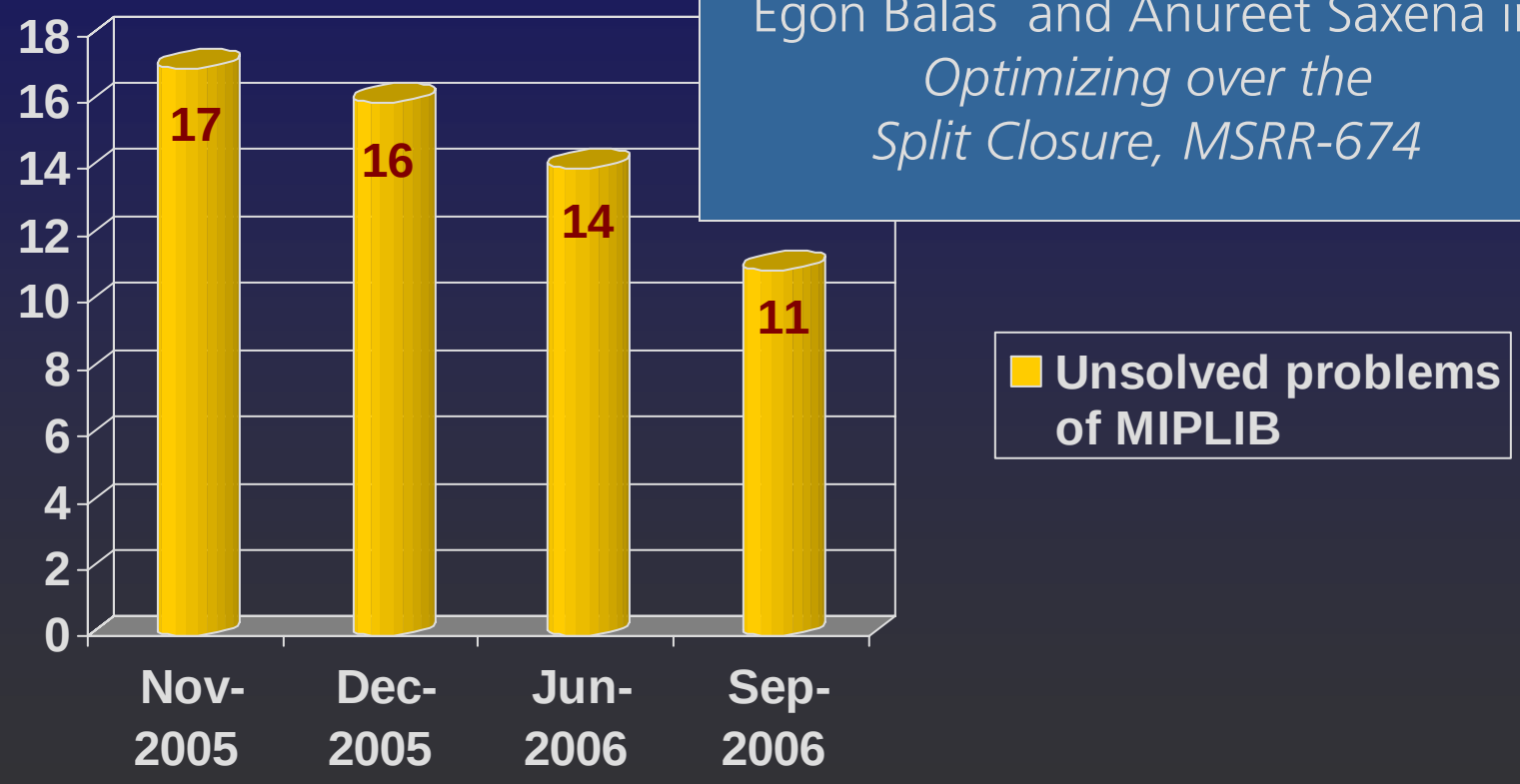
Open Problems of MIPLIB 2003

In Nov. 2005 there were 17 unsolved problems

dash optimization

Open Problems of MIPLIB 2003

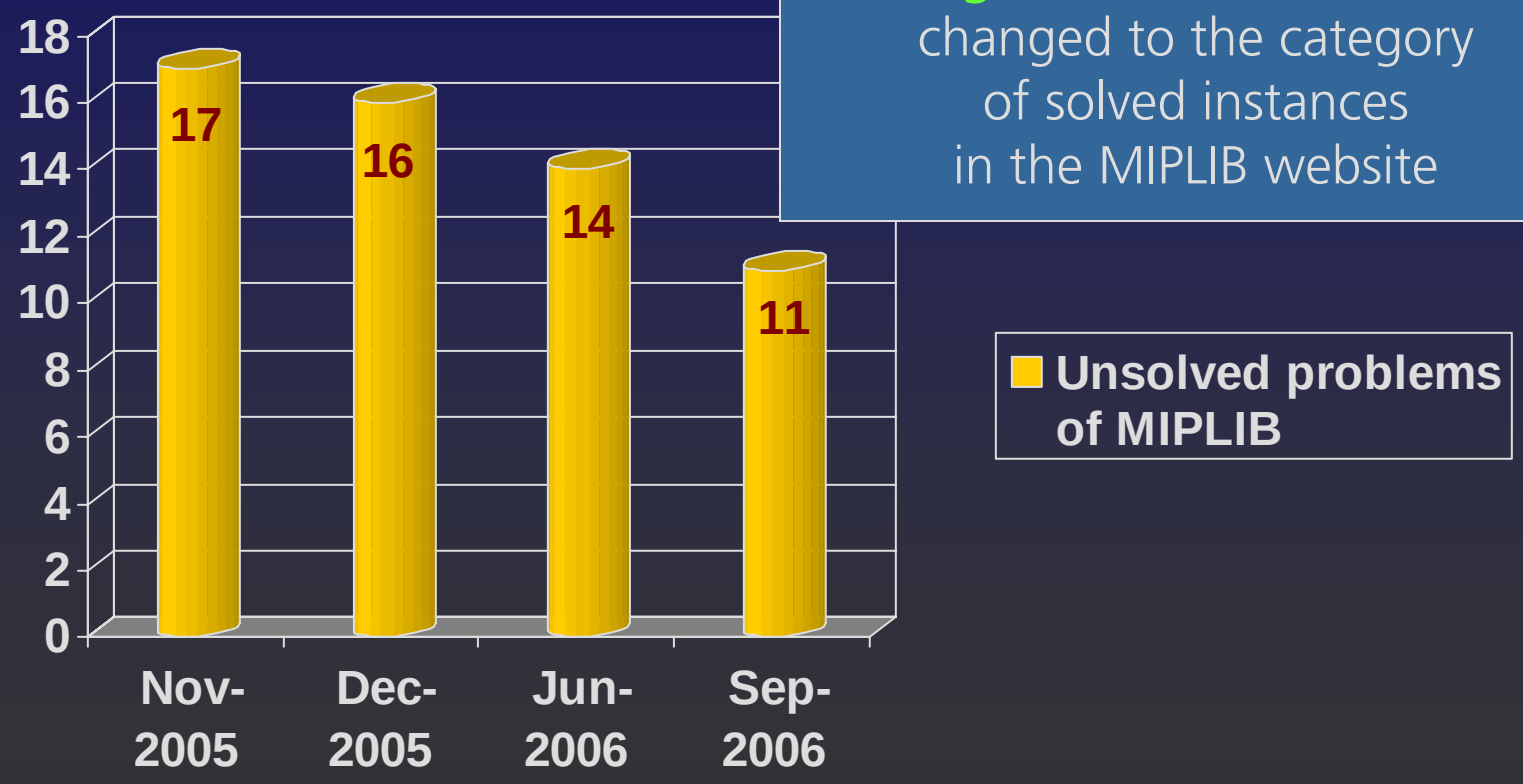
In Dec. 2005, the optimum of **arki001** has been proved by Egon Balas and Anureet Saxena in *Optimizing over the Split Closure, MSRR-674*



dash optimization

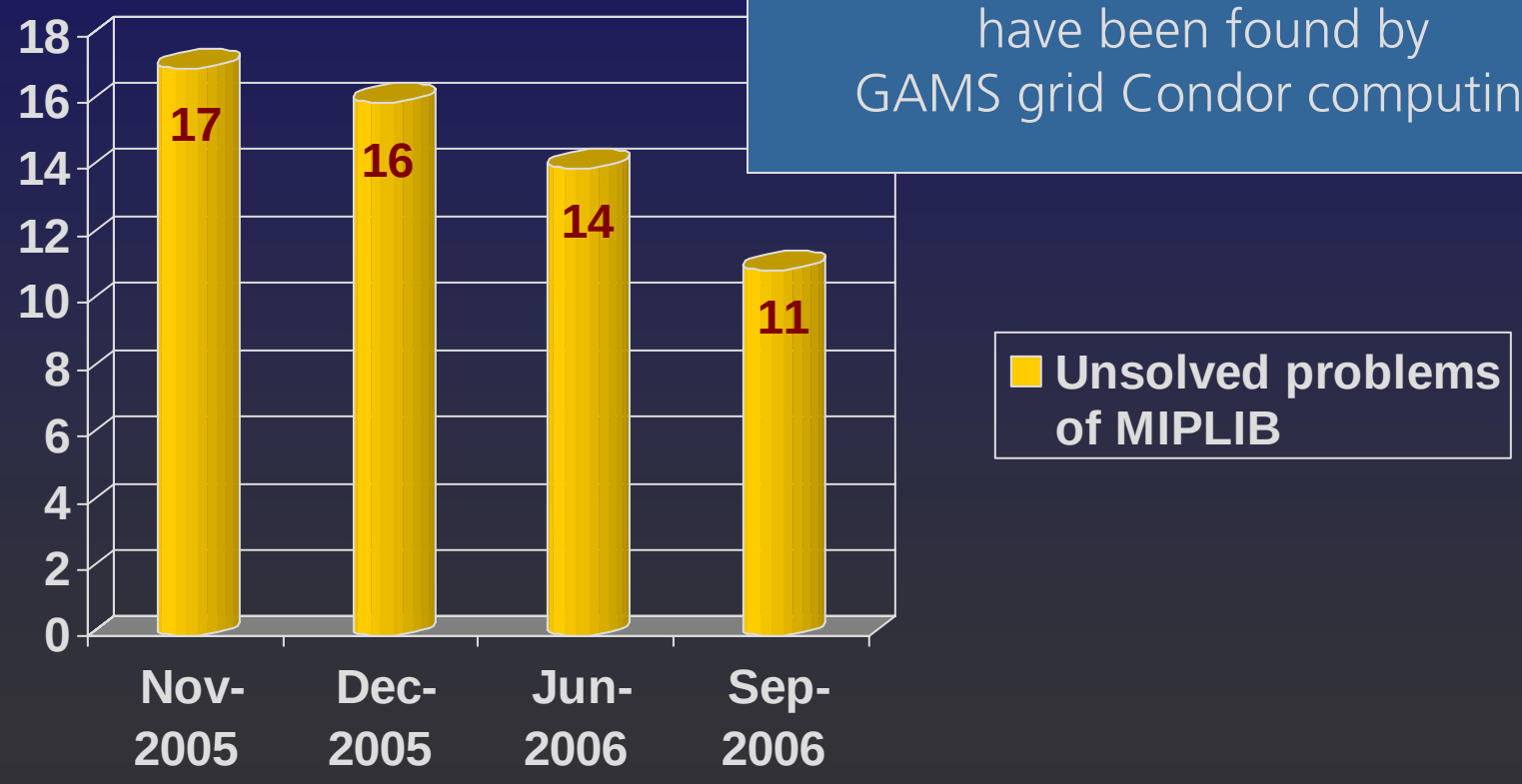
Open Problems of MIPLIB 2003

In June 2006, the status of **glass4** and **roll3000** changed to the category of solved instances in the MIPLIB website



Open Problems of MIPLIB 2003

In Sept. 2006, the optimal values of **a1c1s1**, **timtab2** and **swath** have been found by GAMS grid Condor computing



dash optimization

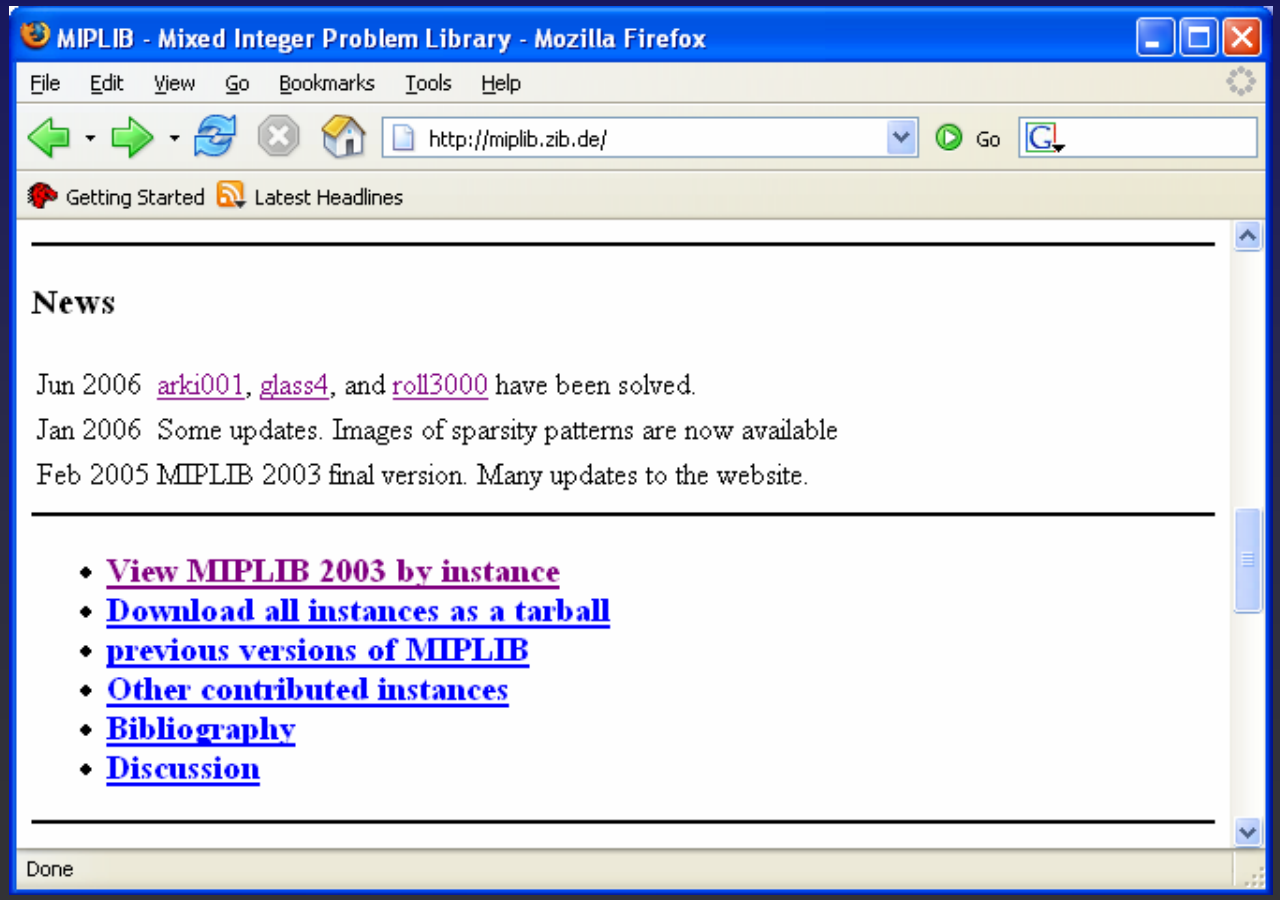
Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs	
arki001	Solved as of Sep-2006
glass4	
roll3000	
a1c1s1	
timtab2	
swath	
atlanta-ip	Unsolved
dano3mip	
ds	
liu	
momentum3	
msc98-ip	
protfold	
rd-rplusc-21	
sp97ar	
stp3d	
t1717	

dash optimization

MIPLIB 2003 Website News

- "Jun 2006: arki001, glass4, and roll3000 have been solved"



arki001

Optimal value is 7580813.046.

Solved using Xpress 2006A in **4 hours**, computing nearly 1,200,000 nodes

Use **no strong branching**.
Use **in-tree cuts** at every node.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP

First solved by Egon Balas and Anureet Saxena, reported in Optimizing over the Split Closure, MSRR-674, December 2005. Solve time was about 64 hours.

arki001

Optimal value is 7580813.046.

Solved using Xpress 2006A in **4 hours**, computing nearly 1,200,000 nodes

Use **no strong branching**.
Use **in-tree cuts** at every node.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP

Tobias Achterberg reports solving the instance using SCIP in below 9 hours, computing nearly 3,000,000 nodes.

arki001

Optimal value is 7580813.046.

Using the optimal value as cutoff,
arki001 is solved by Xpress 2006A in 1.8 hours,
computing nearly 500,000 nodes

First solved by Egon Balas and Anureet Saxena,
reported in Optimizing over the Split Closure, MSRR-674,
December 2005. Solve time was about 64 hours.

glass4

Optimal value is 1200012600.

Solved using Xpress 2006B in **7 hours**.

Use **no cuts**. **Best first** node selection.
Use heavily **Strong Branching**.
Variable selection based on the "up" + "down" pseudo-costs.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP

glass4

Optimal value is 1200012600.

Using the optimal value as cutoff,
glass4 is solved by Xpress 2006B in 1 hour,
computing nearly 1,800,000 nodes

roll3000

Optimal value is 12890.

Solved using Xpress 2006B in **45 min**.

Use **strong branching** heavily.
Put more effort in strong branching at the top nodes.
Use cut generation heavily.
Heavy use of **lift-and-project** cuts at the top node.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP

The MIPLIB website states that roll3000 is
solvable in about **half a day** of computing time.
GAMS grid CONDOR solves it in **50 hours** of CPU time.

roll3000: MIP Parallel Enhancement

Threads	Xpress Time[1] to Solve roll3000 Optimally
1	45 min
2	18 min
Parallel Speedup	2.5x

[1] Xeon 3.0 GHz, 4 GB of RAM, 2 CPU, Hyper-Threading, Windows XP

a1c1s1

Optimal value is 11503.444125.

Solved using Xpress 2006B in **31 hours**.

Use strong branching for a limited number of nodes and then write the leaf nodes out as separate problem files. Solve the resulting problems with a cutoff of 11534.

3GHz Intel Core 2 Duo with 2 threads

GAMS grid CONDOR recently (Sep-06) claimed solving a1c1s1 in **3452 hours** of CPU time.

swath

Optimal value is 467.407491.

Solved using Xpress 2006B in **66 hours**, computing nearly 60,000,000 nodes

Use strong branching for a limited number of nodes and then write the leaf nodes out as separate problem files. Solve the resulting problems with a cutoff of 468.

3GHz Intel Core 2 Duo with 2 threads

GAMS grid CONDOR recently (Sep-06) claimed solving swath in **38,391 hours** of CPU time.

Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs
arki001
glass4
roll3000
a1c1s1
timtab2
swath
atlanta-ip
dano3mip
ds
liu
momentum3
msc98-ip
protfold
rd-rplusc-21
sp97ar
stp3d
t1717

Solved as of Sep-2006

Unsolved

atlanta-ip

Optimal value is 90.0098786144.

Solved using Xpress 2006B in **5.5 hours**.

- ✓ **Do heavy preprocessing**
- ✓ **Split Objective**:
  - Split the objective function into two parts:
  - one with high cost, and another with costs up to 1e-05.
- ✓ **Find feasible solution**:
  - By dropping the low cost component of the objective then it is
  - fairly easy to find a solution that is optimal to within +/- 0.1
- ✓ **Prove Optimality**:
  - By using this near-optimal solution as cutoff and
  - by using heavy cutting and strong branching.

3GHz Intel Core 2 Duo

Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs
arki001
glass4
roll3000
a1c1s1
timtab2
swath
atlanta-ip
dano3mip
ds
liu
momentum3
msc98-ip
protfold
rd-rplusc-21
sp97ar
stp3d
t1717

Solved as of Sep-2006

Solved

Unsolved

dash optimization

msc98-ip

Optimal value is 19839497.005874.

Solved using Xpress 2006B in **1.5 hours**.

Use same model as that used to solve **atlanta-ip**.

3GHz Intel Core 2 Duo

Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs	
arki001	Solved as of Sep-2006
glass4	
roll3000	
a1c1s1	
timtab2	
swath	
atlanta-ip	Solved
msc98-ip	
ds	Unsolved
liu	
momentum3	
dano3mip	
protfold	
rd-rplusc-21	
sp97ar	
stp3d	
t1717	

dash optimization

rd-rplusc-21

Optimal value is 165395.2753.

Solved using Xpress 2006B in **1.7 hours**.

The key is to find good estimates for
variable selection (using heavy strong branching)

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP

rd-rplusc-21

Optimal value is 165395.2753.

Using 165,500 as cutoff value,
and constraint branching
rd-rplusc-21 is solved by Xpress 2006B
in 10 min (14 min if no cutoff is used),
computing nearly 80,000 nodes

Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs	
arki001	Solved as of Sep-2006
glass4	
roll3000	
a1c1s1	
timtab2	
swath	
atlanta-ip	Solved
msc98-ip	
rd-rplusc-21	
ds	Unsolved
liu	
momentum3	
dano3mip	
protfold	
sp97ar	
stp3d	
t1717	

dash optimization

MIPLIB 2003: New Xpress Improved Solutions

Problem	Old Best Known Obj. Value (*)	Xpress Improved Obj. Value (**)	GAIN (|1-(**)/(*)|)
atlanta-ip	95.009549704	90.00987861	5.3%
msc98-ip	20980991.006	19839497.006	5.4%
rd-rplusc-21	171182	165395.2753	3.4%
ds	283.4425	116.59	58.9%
momentum3	370177.036	236426.335	36.1%
t1717	193221	170195	11.9%
protfold	-30	-31	3.3%
liu	1172	1138	2.9%
dano3mip	691.2	687.733333	0.5%
sp97ar	664565103.76	661670441.4	0.4%

Optimal

Unsolved

Xpress found 1st Feasible Solution of stp3d

- Using a maximum degradation strategy with fast node re-optimizations the first feasible solution of **stp3d** has been found in 5 hours by Xpress

stp3d first feasible solution has value **529.778190**

- Subsequently this solution has been improved using the new local search heuristics of Xpress

stp3d best solution has value **500.736**

Xpress found 1st Feasible Solution of stp3d

- Using a maximum degradation strategy with fast node re-optimizations the first feasible solution of

The 500.736 solution is within 3.3% of the stp3d optimal value

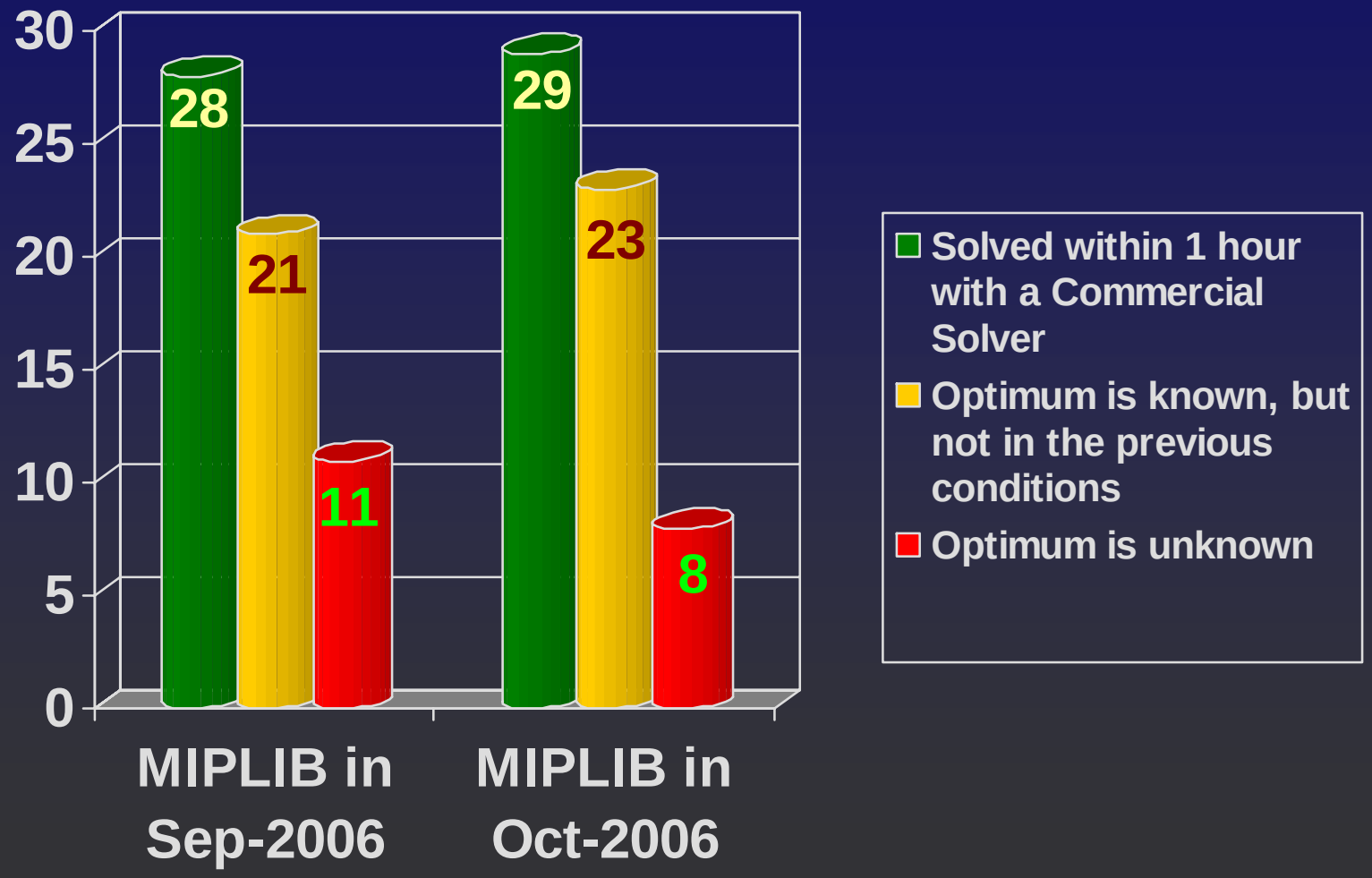
stp3d best solution has value **500.736**

dash optimization

MIPLIB 2003: Closing the Gap of Unsolved Cases

Problem	Upper Bound (**)	Lower Bound (*)	GAP (|1-(**)/(*)|)
sp97ar	661670441.4	657862912	0.6%
protfold	-31	-32	3.1%
stp3d	500.736	484.71817	3.3%
dano3mip	687.733333	578.05603	19.0%
t1717	170195	136538.4219	24.6%
ds	116.59	76.32504272	52.8%
liu	1138	560	103.2%
momentum3	236426.335	94824.16406	149.3%

Current Status of MIPLIB 2003



dash optimization

Xpress MIP Components that Lead to the MIPLIB Improvements

- Better Presolving
- Better Cutting
  - Use of Lift-and-Project Cuts
- Better Heuristics
  - Improved Local Search Heuristics
- Better Branching Decisions
  - Improved Strong Branching Estimates

Final Remarks

- Xpress 2006B
  - Solved 8 (out of 17) problems unsolved before Dec-2005
  - Solved 3 (5 if a single computer is considered) new problems for the 1st time: atlanta-ip, msc98-ip and rd-rplusc-21 (a1c1s1 and swath)
  - Found the 1st feasible solution for the stp3d problem
  - Found better solutions for all the remaining 7 unsolved problems

dash optimization