

# MINTB: MINimize the number of Toll Booths

## Problem description

MINTB is a mixed-integer linear program (in which the integer variables are all binary). It somewhat resembles a hybrid of a multicommodity flow model and a set covering problem. The setting is a road network, represented as a digraph with multiple traffic types (“commodities”) based on the combination of entry and exit node used by the traffic. Travel time on any arc is affected by congestion, according to a stipulated function (that is external to the MINTB model). Planners seek to manipulate the equilibrium flow resulting from drivers optimizing their individual routes into conforming to an overall system optimum (minimum total travel time across all drivers) by adding tolls to certain congested arcs.

Computation of both the user and system equilibrium flows is external to the MINTB model. MINTB simultaneously selects a minimal set of arcs to toll and the necessary (positive) tolls on those arcs. Variables consist of a set of binary variables (one per arc) selecting arcs to toll, a set of nonnegative variables (one per arc) selecting the tolls to charge, and a set of free variables (one for every combination of a commodity and an arc) which are KKT multipliers from an external nonlinear model.

The constraints fall into two categories. The bulk of the constraints are inequalities relating the tolls to the KKT multipliers. Satisfaction of those inequalities is necessary and sufficient to ensure that the user equilibrium flow with those tolls will match the system optimal solution. The remaining constraints, of the form  $toll_{ij} \leq M * select_{ij}$  ties the toll charge variables (*toll*) to the binary variables selecting the arcs to toll (*select*) such that an arc can only be tolled if it has been selected (and counted in the objective).

One set of inequality constraints is restricted to combinations of commodities and arcs for which the (externally computed) system optimal solution has a positive flow. The magnitudes of those flows can vary greatly, and in practice it may prudent to consider small positive flows as being zero to within rounding error (removing the corresponding inequalities from MINTB). In the ZIMPL model file, this is accomplished by setting the parameter `minFlow` to the threshold for counting a flow as positive. The value specified in the file, 0.001, was used in the *Operations Research* paper.

## Files Supplied

Three scenarios based on real-world road networks are included, of which one (Stockholm) is challenging to solve.

**mintb.zpl** A ZIMPL model file. The parameter `datafile` contains the name of the data file to use; the default, Stockholm, is the hardest of the three scenarios provided but can be overridden to process one of the other two scenarios.

**siouxfalls.zdat** Data for the Sioux Falls (USA) scenario, which is easy to solve.

**hull.zdat** Data for the Hull (Canada) scenario, which is fairly easy to solve.

**stockholm.zdat** Data for the Stockholm scenario, which is unsolved as of the data of submission.

**stockholm.mps** MPS model file for the Stockholm scenario.

## Problem Characteristics

City	Nodes	Arcs	Commodities	$M$	Best Solution	Best Bound
Sioux Falls	24	76	24	1	32	32
Hull	446	798	16	10	39	39
Stockholm	416	962	45	1000	133	102.3

Values for  $M$  are based on knowledge of the optimal tolls in the cases of Sioux Falls and Hull (and may be tighter than what would normally be safe to select); the value of  $M$  for Stockholm is based on knowledge of a rather weak feasible solution.

## References

```
@INPROCEEDINGS{Bai2006,  
  author = {Lihui Bai and Matthew T. Stamps and R. Corban Harwood and Christopher J. Ko},  
  title = {A Genetic Algorithm for the Minimum Tollbooth Problem},  
  booktitle = {Proceedings of the 2006 Meeting of the Decision Sciences Institute},  
  year = {2006}  
}  
@ARTICLE{Bai2009,  
  author = {Lihui Bai and Paul A. Rubin},  
  title = {Combinatorial Benders Cuts for the Minimum Tollbooth Problem},  
  journal = {Operations Research},  
  year = {2009},  
  volume = {57},  
  pages = {1510--1522},  
  number = {6},  
  month = {November-December}  
}
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