dash optimization

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Outline

- LP benchmarks
- Xpress performance on MIPLIB 2003
- Conclusions
Barrier Algorithm
(Mittelmann Benchmark for LP)

Computing Times (in sec)

2006B is 12% faster than 2005B on an “average” case

2006B is 27% faster than 2005B on the “larger” problems

NOTE: The computing times include the (crossover) time needed to find a basic feasible solution. Considers 35 LPs.
A **quartile** is any of the three values which divide the sorted data set into four equal parts, so that each part represents $1/4^{th}$ of the sample

- **1st quartile**
  - lower quartile = cuts off lowest 25% of data = 25th percentile
- **2nd quartile**
  - median = cuts data set in half = 50th percentile
- **3rd quartile**
  - upper quartile = cuts off highest 25% of data, or lowest 75% = 75th percentile
Dual Algorithm
(Mittelmann Benchmark for LP)

Computing Times (in sec)

2006B is 12% faster than 2005B on an "average" case

2006B is 70% faster than 2005B on the "larger" problems

NOTE: Considers 34 LPS.
Primal Algorithm
(Mittelmann Benchmark for LP)

2006B is 11% faster than 2005B on an "average" case

2006B is 7% faster than 2005B on the "larger" problems

NOTE: Considers 34 LPs.
### Xpress LP Improvements

(Mittelmann Benchmark for LP)

<table>
<thead>
<tr>
<th>LP Algorithms</th>
<th>Geometric Mean</th>
<th>3rd Quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier</td>
<td>12% faster</td>
<td>27% faster</td>
</tr>
<tr>
<td>Dual</td>
<td>12% faster</td>
<td>70% faster</td>
</tr>
<tr>
<td>Primal</td>
<td>11% faster</td>
<td>7% faster</td>
</tr>
</tbody>
</table>
MIPLIB 2003 consists of 60 MIPs.

According to the MIPLIB website (Oct-2006):

- **Solved within 1 hour with a Commercial Solver**
- **Optimum is known, but not in the previous conditions**
- **Optimum is unknown**

MIPLIB 2003 problems:
- 28 problems solved within 1 hour
- 18 problems with optimum known in previous conditions
- 14 problems with optimum unknown
Xpress Performance on MIPLIB 2003

- Xpress Improvements in the Group of “Simpler” Problems

- Xpress Improvements in the Group of “Challenging” Problems
MIPLIB 2003: Optimum within 30’ with Default Settings

Number of MIPLIB problems solved optimally by Xpress within 30 minutes

(1) Pentium 4 3.6 GHz, 2 GB of RAM, Hyper-Threading, Windows XP
In 30 min, Xpress 2006B solves 27 “simple” problems (according to the MIPLIB website) + harp2, which can be solved by Xpress 2006B in 1 minute\(^{(1)}\) with default settings

\(^{(1)}\) Pentium 4 3.6 GHz, 2 GB of RAM, Hyper-Threading, Windows XP
Open Problems of MIPLIB 2003

In Nov. 2005 there were 17 unsolved problems
Open Problems of MIPLIB 2003

In Dec. 2005, the optimum of arki001 has been proved by Egon Balas and Anureet Saxena in Optimizing over the Split Closure, MSRR-674.

Unsolved problems of MIPLIB
In June 2006, the status of glass4 and roll3000 changed to the category of solved instances in the MIPLIB website.
In Sept. 2006, the optimal values of \texttt{a1c1s1}, \texttt{timtab2} and \texttt{swath} have been found by GAMS grid Condor computing.
Open Problems of MIPLIB 2003 before Dec-2005

- arki001
- glass4
- roll3000
- a1c1s1
- timtab2
- swath
- atlanta-ip
- dano3mip
- ds
- liu
- momentum3
- msc98-ip
- protfold
- rd-rplusc-21
- sp97ar
- stp3d
- t1717

17 MIPs Solved as of Sep-2006

Unsolved
“Jun 2006: arki001, glass4, and roll3000 have been solved”
First solved by Egon Balas and Anureet Saxena, reported in Optimizing over the Split Closure, MSRR-674, December 2005. Solve time was about 64 hours.

Optimal value is 7580813.046.

Solved using Xpress 2006A in **4 hours**, computing nearly **1,200,000 nodes**

Use **no strong branching**.
Use **in-tree cuts** at every node.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading, 4 GB of RAM, Windows XP
Tobias Achterberg reports solving the instance using SCIP in below 9 hours, computing nearly 3,000,000 nodes.

Solved using Xpress 2006A in 4 hours, computing nearly 1,200,000 nodes.

Use no strong branching.
Use in-tree cuts at every node.

Optimal value is 7580813.046.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading, 4 GB of RAM, Windows XP

Tobias Achterberg reports solving the instance using SCIP in below 9 hours, computing nearly 3,000,000 nodes.
Using the optimal value as cutoff, arki001 is solved by Xpress 2006A in 1.8 hours, computing nearly 500,000 nodes.
Solved using Xpress 2006B in 7 hours.

Optimal value is 1200012600.

Use no cuts. Best first node selection.
Use heavily Strong Branching.
Variable selection based on the “up” + “down” pseudo-costs.

Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
4 GB of RAM, Windows XP
Solved using Xpress 2006B in 7 hours.

Use no cuts.
Best first node selection.
Use heavily Strong Branching.
Variable selection based on the "up" + "down" pseudo-costs.

Optimal value is 1200012600.

Using the optimal value as cutoff, glass4 is solved by Xpress 2006B in 1 hour, computing nearly 1,800,000 nodes.
optimal value is 12890.

\begin{itemize}
\item \textbf{Solved using Xpress 2006B in 45 min.}
\item Use \textbf{strong branching} heavily.
\item Put more effort in strong branching at the top nodes.
\item Use cut generation heavily.
\item Heavy use of \textbf{lift-and-project} cuts at the top node.
\end{itemize}

\begin{itemize}
\item Xeon 3.0 GHz, 2 CPU, Hyper-Threading,
\item 4 GB of RAM, Windows XP
\end{itemize}

The MIPLIB website states that roll3000 is solvable in about \textbf{half a day} of computing time.

GAMS grid CONDOR solves it in \textbf{50 hours} of CPU time.
## roll3000: MIP Parallel Enhancement

<table>
<thead>
<tr>
<th>Threads</th>
<th>Xpress Time(^{(1)}) to Solve roll3000 Optimally</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>45 min</td>
</tr>
<tr>
<td>2</td>
<td>18 min</td>
</tr>
<tr>
<td>Parallel Speedup</td>
<td>2.5x</td>
</tr>
</tbody>
</table>

\(^{(1)}\) Xeon 3.0 GHz, 4 GB of RAM, 2 CPU, Hyper-Threading, Windows XP
Solved using Xpress 2006B in 31 hours.

Optimal value is 11503.444125.

Use strong branching for a limited number of nodes and then write the leaf nodes out as separate problem files. Solve the resulting problems with a cutoff of 11534.

3GHz Intel Core 2 Duo with 2 threads

GAMS grid CONDOR recently (Sep-06) claimed solving a1c1s1 in 3452 hours of CPU time.
Optimal value is 467.407491.

Solved using Xpress 2006B in 66 hours, computing nearly 60,000,000 nodes.

Use strong branching for a limited number of nodes and then write the leaf nodes out as separate problem files. Solve the resulting problems with a cutoff of 468.

3GHz Intel Core 2 Duo with 2 threads

GAMS grid CONDOR recently (Sep-06) claimed solving swath in 38,391 hours of CPU time.
Open Problems of MIPLIB 2003 before Dec-2005

<table>
<thead>
<tr>
<th>Unsolved</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 MIPs</td>
</tr>
<tr>
<td>arki001</td>
</tr>
<tr>
<td>glass4</td>
</tr>
<tr>
<td>roll3000</td>
</tr>
<tr>
<td>a1c1s1</td>
</tr>
<tr>
<td>timtab2</td>
</tr>
<tr>
<td>swath</td>
</tr>
<tr>
<td>atlanta-ip</td>
</tr>
<tr>
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<tr>
<td>ds</td>
</tr>
<tr>
<td>liu</td>
</tr>
<tr>
<td>momentum3</td>
</tr>
<tr>
<td>msc98-ip</td>
</tr>
<tr>
<td>protfold</td>
</tr>
<tr>
<td>rd-rplusc-21</td>
</tr>
<tr>
<td>sp97ar</td>
</tr>
<tr>
<td>stp3d</td>
</tr>
<tr>
<td>t1717</td>
</tr>
</tbody>
</table>

Solved as of Sep-2006
atlanta-ip

Optimal value is 90.0098786144.

Solved using Xpress 2006B in **5.5 hours**.

- **Do heavy preprocessing**
- **Split Objective:**
  - Split the objective function into two parts:
  - one with high cost, and another with costs up to 1e-05.
- **Find feasible solution:**
  - By dropping the low cost component of the objective then it is
  - fairly easy to find a solution that is optimal to within +/- 0.1
- **Prove Optimality:**
  - By using this near-optimal solution as cutoff and
  - by using heavy cutting and strong branching.

**3GHz Intel Core 2 Duo**
Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs
- arki001
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- sp97ar
- stp3d
- t1717

Solved as of Sep-2006

Solved

Unsolved
msc98-ip

Optimal value is 19839497.005874.

Solved using Xpress 2006B in 1.5 hours.

Use same model as that used to solve atlanta-ip.

3GHz Intel Core 2 Duo
Open Problems of MIPLIB 2003 before Dec-2005

17 MIPs
- arki001
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Solved as of Sep-2006
- Solved
- Unsolved
rd-rplusc-21

Optimal value is 165395.2753.

Solved using Xpress 2006B in 1.7 hours.

The key is to find good estimates for variable selection (using heavy strong branching)

Xeon 3.0 GHz, 2 CPU, Hyper-Threading, 4 GB of RAM, Windows XP
The key is to find good estimates for variable selection (using heavy strong branching).

Optimal value is 165395.2753.

Using 165,500 as cutoff value, and constraint branching, rd-rplusc-21 is solved by Xpress 2006B in 10 min (14 min if no cutoff is used), computing nearly 80,000 nodes.
### Open Problems of MIPLIB 2003 before Dec-2005

<table>
<thead>
<tr>
<th>17 MIPs</th>
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<tbody>
<tr>
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</tr>
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<td>timtab2</td>
</tr>
<tr>
<td>swath</td>
</tr>
<tr>
<td><strong>atlanta-ip</strong></td>
</tr>
<tr>
<td><strong>msc98-ip</strong></td>
</tr>
<tr>
<td><strong>rd-rplusc-21</strong></td>
</tr>
<tr>
<td>ds</td>
</tr>
<tr>
<td>liu</td>
</tr>
<tr>
<td><strong>momentum3</strong></td>
</tr>
<tr>
<td>dano3mip</td>
</tr>
<tr>
<td>protfold</td>
</tr>
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</tr>
<tr>
<td>stp3d</td>
</tr>
<tr>
<td>t1717</td>
</tr>
</tbody>
</table>

Solved as of Sep-2006

Solved

Unsolved
## MIPLIB 2003:
New Xpress Improved Solutions

<table>
<thead>
<tr>
<th>Problem</th>
<th>Old Best Known Obj. Value (*)</th>
<th>Xpress Improved Obj. Value (**)</th>
<th>GAIN (1-(**)/*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>atlanta-ip</td>
<td>95.009549704</td>
<td>90.00987861</td>
<td>5.3%</td>
</tr>
<tr>
<td>msc98-ip</td>
<td>20980991.006</td>
<td>19839497.006</td>
<td>5.4%</td>
</tr>
<tr>
<td>rd-rplusc-21</td>
<td>171182</td>
<td>165395.2753</td>
<td>3.4%</td>
</tr>
<tr>
<td>ds</td>
<td>283.4425</td>
<td>116.59</td>
<td>58.9%</td>
</tr>
<tr>
<td>momentum3</td>
<td>370177.036</td>
<td>236426.335</td>
<td>36.1%</td>
</tr>
<tr>
<td>t1717</td>
<td>193221</td>
<td>170195</td>
<td>11.9%</td>
</tr>
<tr>
<td>protfold</td>
<td>-30</td>
<td>-31</td>
<td>3.3%</td>
</tr>
<tr>
<td>liu</td>
<td>1172</td>
<td>1138</td>
<td>2.9%</td>
</tr>
<tr>
<td>dano3mip</td>
<td>691.2</td>
<td>687.7333333</td>
<td>0.5%</td>
</tr>
<tr>
<td>sp97ar</td>
<td>664565103.76</td>
<td>661670441.4</td>
<td>0.4%</td>
</tr>
</tbody>
</table>
Xpress found 1st Feasible Solution of stp3d

- Using a maximum degradation strategy with fast node re-optimizations the first feasible solution of stp3d has been found in 5 hours by Xpress.

  stp3d first feasible solution has value 529.778190

- Subsequently this solution has been improved using the new local search heuristics of Xpress.

  stp3d best solution has value 500.736
Xpress found 1\textsuperscript{st} Feasible Solution of stp3d

Using a maximum degradation strategy with fast node re-optimizations the first feasible solution of stp3d has been found in 5 hours by Xpress.

The 500.736 solution is within 3.3\% of the stp3d optimal value.

stp3d best solution has value 500.736
MIPLIB 2003: Closing the Gap of Unsolved Cases

| Problem    | Upper Bound (***) | Lower Bound (*) | GAP $|1-(***)/(*)|$ |
|------------|--------------------|-----------------|-----------------|
| sp97ar     | 661670441.4        | 657862912       | 0.6%            |
| protfold   | -31                | -32             | 3.1%            |
| stp3d      | 500.736            | 484.71817       | 3.3%            |
| dano3mip   | 687.7333333        | 578.05603       | 19.0%           |
| t1717      | 170195             | 136538.4219     | 24.6%           |
| ds         | 116.59             | 76.32504272     | 52.8%           |
| liu        | 1138               | 560             | 103.2%          |
| momentum3  | 236426.335         | 94824.16406     | 149.3%          |
Current Status of MIPLIB 2003

- **MIPLIB in Sep-2006**
  - Solved within 1 hour with a Commercial Solver: 28
  - Optimum is known, but not in the previous conditions: 21
  - Optimum is unknown: 11

- **MIPLIB in Oct-2006**
  - Solved within 1 hour with a Commercial Solver: 29
  - Optimum is known, but not in the previous conditions: 23
  - Optimum is unknown: 8

Legend:
- Green: Solved within 1 hour with a Commercial Solver
- Yellow: Optimum is known, but not in the previous conditions
- Red: Optimum is unknown
Xpress MIP Components that Lead to the MIPLIB Improvements

- Better Presolving
- Better Cutting
  - Use of Lift-and-Project Cuts
- Better Heuristics
  - Improved Local Search Heuristics
- Better Branching Decisions
  - Improved Strong Branching Estimates
Final Remarks

- **Xpress 2006B**
  - Solved 8 (out of 17) problems unsolved before Dec-2005
  - Solved 3 (5 if a single computer is considered) new problems for the 1st time: atlanta-ip, msc98-ip and rd-rplusc-21 (a1c1s1 and swath)
  - Found the 1st feasible solution for the stp3d problem
  - Found better solutions for all the remaining 7 unsolved problems
dash optimization