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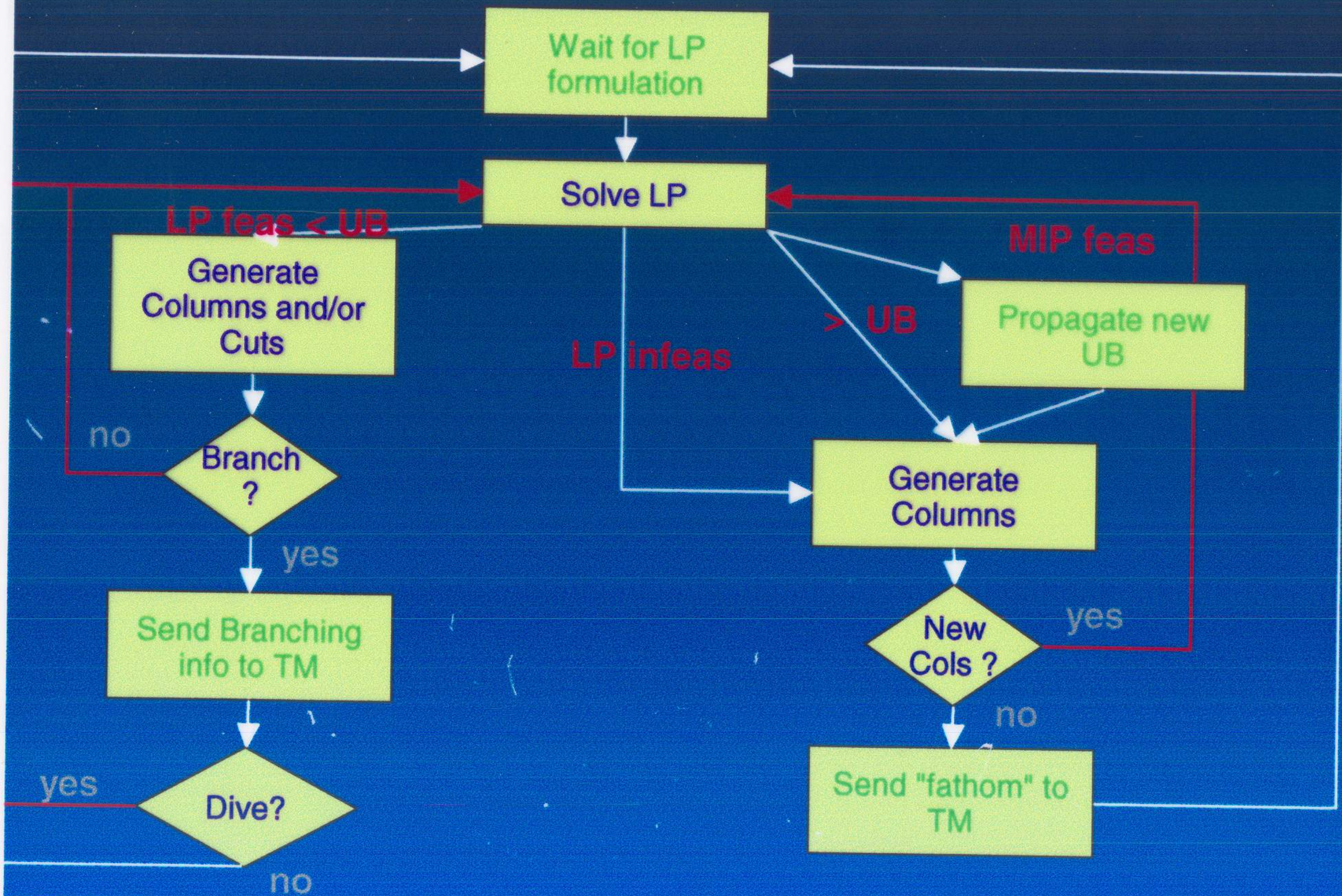
Experience with a Parallel Branch, Cut and Price framework

Features

- Generalized Branching objects:
  - Change bounds of variables/constraints
  - even $y_i = 0$, $\Sigma a_j x_j \leq b$ or $y_i = 1$
- Use of Cut and Variable pools:
  - Locally valid cuts (Globally valid as special case)
  - Variable pool
- User "Hooks" (mostly C++ virtual objects)
  - LP Solver Class
    - ✓Solve LPs
    - ✓Add/Delete Rows/Columns
    - ✓Modify bounds
    - ✓Return/accept warmstart information
  - Message Passing Protocol
  - Cut/Variable generation
  - Branching Object generation
  - Logical fixing
  - ....

Flow at LP node



Overview

- LP formulation based:
  - Lower bounding by solving LPs
  - Any LP solver (not necessarily Simplex based)
- Parallel:
  - Master / Slaves model.
  - Distributed Network.
- Branch
- and Cut (constraint generation):
  - Strengthen LP formulation
  - Dynamically use constraints in subproblems.
- and Price (variable generation):
  - Include new variables in LP formulation
  - Dynamically use variables in subproblems.
- Framework:
  - User has to provide/select problem specific parts.

swath

- **Initial statistics:**
  - 885 rows, 6805 variables of which 6724 0-1, 1 for GAMS objective and 80 continuous.
  - Continuous solution 334.4968581, best known integer solution 497.603.
- **Initial analysis:**
  - 80 continuous come in 20 groups of 4 identical variables - so we can reduce to 20 variables - x_i
  - We can introduce 380 y accumulation variables so $y_{ij} = \Sigma z_{ijk}$ (but $\Sigma z_{ijk} \leq 1$ so y_{ij} a 0-1 variable.
  - x_i have no cost

swath

- n=20
- $20\, y_{ij} + x_i - x_j \leq 19$ for i=0…19, j=0…19 i!=j
- where y_{ij} are 0-1 and x_i are "continuous"
- these are only constraints where x appears
- not only are y 0-1 but $\Sigma\, y_{ij} \leq 1$ for all i
- If $x_j \geq x_i+1$ then y_{ij} can be 1 (and y_{ji} is 0)
- If smallest x is >0 then all x can be adjusted.
- Given order of x, values of x can be adjusted so that x takes on values 0…19 exactly once.

- So x variables are really general integer and getting them correct forces correct solution!

swath

- $20\ y_{ij} + x_i - x_j \leq 19$
- $20\ y_{ji} + x_j - x_i \leq 19$

- $y_{ij} + y_{ji} \leq 1$

- for any i,j,k
- $y_{ij} + y_{ji} \leq 1$, $y_{ik} + y_{ki} \leq 1$, $y_{jk} + y_{kj} \leq 1$
- $y_{ij} + y_{ik} \leq 1$, $y_{ji} + y_{jk} \leq 1$, $y_{ki} + y_{kj} \leq 1$
- Only way three y can be one is $y_{ij} = y_{jk} = y_{ki} = 1$ or similar ->
- $x_j \geq x_i + 1$, $x_k \geq x_j + 1$, $x_i \geq x_k + 1$ ->
- $x_i \geq x_i + 3$!! - so ->
- $y_{ij} + y_{ji} + y_{jk} + y_{kj} + y_{ki} + y_{ik} \leq 2$

swath

- Original:
  - Continuous solution 334.4968581, best known solution 497.603.
- Previous cuts can be generalized
- With cuts and reformulation and y_{ij} in SOS
  - Continuous 461.8312
  - Proven best solution 467.4075
  - 448 nodes without any further cuts
- Key ideas
  - Simplify so can see structure
  - Deduce x relationships
  - Use for powerful cuts

seymour

- Set covering problem
- Initial statistics:
  - 4,944 rows and 1,372 columns.
  - All 0-1 , objective is all 1.0
  - Continuous solution 403.8465, best known solution 423.
- Initial analysis:
  - Can be reduced to 4,323 rows and 882 columns.

seymour

- Cuts
  - Odd hole cuts
  - Prime cover cuts (Bellmore-Ratcliff)
  - .....
  - [Disjunctive cuts] - should have tried
- Branching
  - Close to half (and long columns)
  - $x_i = x_j = 1$ or $x_i + x_j \leq 1$
  - Slack branching cut
  - On tight constraints - $x_1 = 1$ or $x_1 = 0$ & $x_2 = 1$ or ..
  - ......
- Parallel on 30 machines - was obvious would take too long.
  - We intend to keep trying

mkc

- Initial statistics:
  - 3411 rows, 5325 variables of which 5323 0-1.
  - Continuous solution -611.85, best known solution -553.75.
- IBM solution for steel mill planning
- Initial analysis(RC 21071):
  - $\Sigma_{i \varepsilon N j} O^i x_j^i \le W_j z_j \qquad 1 \le j \le M$
  - $\Sigma_{j \varepsilon N i} x_j^i \le 1 \qquad 1 \le i \le N$
  - $\Sigma_{c \varepsilon C j} y_j^c \le 2 \qquad 1 \le j \le M$
  - $x_j^i \le y_j^{c(i)} \qquad 1 \le i \le N, \quad 1 \le j \le M$
- M = 24 (slabs) and N = 439 (orders)
- Knapsacks do not overlap

mkc

- **Column Generation approach:**
  - Master has 2 continuous variables (purely for reporting) and 24 z variables
  - 24 ex-knapsack constraints $\Sigma_{q \varepsilon Q j}\ q_{qj} \le z_j$
  - 439 constraints $\Sigma_{j \varepsilon N i}\ \delta_{qij}\, q_{qj} \le 1$ where δ_{qij} is 1 if x_{ij} is included in q_{qj}

- **Each knapsack**
  - $\Sigma_{i \varepsilon N j}\ O^i x_j^i \le W_j$
  - $\Sigma_{c \varepsilon C j}\ y_j^c \le 2$
  - $x_j^i \le y_j^{c(i)}$ $\qquad 1 \le i \le N$

mkc

- Brute force is possible:
  - Number of x variables in a knapsack varies from 7 to 290.
  - Total number of q variables is only 604,333
- Results:
  - Total generation and solution time 4 seconds. Integer LB is -563.846 as against original of -611.85.
  - Continuous solution is integer so proven optimal solution of -563.846
- Bad/good news:
  - IBM problem - customer says - well that was only a toy problem - can you solve this?

mkc7

- Statistics:
  - 68,122 rows, 62,394 variables of which 62,392 0-1.
  - Continuous solution -1200.947
  - Column generation master 9,560 rows and initial 0-1 solution of 0.0
- M = 74 and N = 9,483
- Not quite as easy!
  - Knapsacks still do not overlap
  - Some y variables switch on 700 x variables and 700!10 proposals in some knapsacks

mkc7

- Approach:
  - Most large knapsacks
    - ✓ Use simple greedy heuristic
    - ✓ Close to LP relaxation
  - Small knapsacks
    - ✓ Enumerate once and keep in pool
  - Remaining knapsacks
    - ✓ Use simple heuristic - but
    - ✓ Not close to LP relaxation - needs better heuristics
  - Looks as if gives close to optimal answers - more work needed on getting LB.

mkc7 - results

Effect of full enumeration of knapsacks on solution quality

enumerate if size of knapsack	variables generated	Best possible LB	Best LP solution found	Best IP solution found	Time to best IP (seconds)
<1	0	-1186.115	-1177.489	-1160.723	34.31
<30	1678	-1186.038	-1177.685	-1171.171	77.48
<50	104723	-1185.968	-1182.967	-1178.163	70.81

Original formulation had LB of -1200.947
(and took 101 seconds to continuous optimum!)

- Two problems solved - third by end year?
- Column generation
  - Not totally symmetric with cut generation (especially when exact continuous optimum can not be found)
  - Theoretically can do cuts and price on same problem
  - Good to get solutions early
  - Need column generation in parallel in early phases
- Flexible branching objects
  - Mixed variable/constraint branching
  - Easier valid branching with column generation
- Thinking beats computing power
- Exploring ways to make framework more available